Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 97112

B.E./B.Tech. DEGREE EXAMINATION, DECEMBER 2015/JANUARY 2016.

First Semester

Civil Engineering

MA 1101 – MATHEMATICS – I

(Common to all branches)

(Regulation 2008)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 3 & 4 \\ 0 & 0 & 5 \end{bmatrix}$ then find the eigen values of A^{-1}.

2. Find the quadratic form associated with the symmetric matrix $A = \begin{bmatrix} 2 & 5/2 & -1 \\ 5/2 & 3 & 3/2 \\ -1 & 3/2 & 4 \end{bmatrix}$

3. Find the angle between the straight lines $\frac{x}{2} = \frac{y}{-2} = \frac{z}{1}$ and $\frac{x-4}{2} = \frac{y-5}{1} = \frac{z+6}{2}$.

4. Find the equation of the sphere passing through the circle given by $x^2 + y^2 + z^2 + 3x + y + 4z - 3 = 0$ and $x^2 + y^2 + z^2 + 2x + 3y + 6 = 0$ and the point $(1, -2, 3)$.

5. Find the radius of curvature of the curve $y^2 = x^3$.

6. Define the envelope of the family of curves.

7. If $u = x^3y^3 + x^2y^3$ and $x = t^2$, $y = 2t$ then find $\frac{du}{dt}$ without subsituting x and y in u.

8. If $u = x^2 - y^2$, $v = 2xy$ evaluate $\frac{\partial(x, y)}{\partial(u, v)}$.

9. Find the particular integral of $(D^2 - 4D + 3)y = x^2$.

10. Convert the differential equation $(x^2D^2 + xD + 1)y = \sin(2\log x)\sin(\log x)$ into an equation having constant coefficients.

PART B — (5 × 16 = 80 marks)

11. (a) Reduce the quadratic form $8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 - 8x_2x_3 + 4x_3x_1$ into the canonical form through an orthogonal transformation. Write down the orthogonal transformation, which you use. Find the nature, rank, index and signature of the quadratic form. (16)

Or

(b) (i) Solve $(2x+3)^2 y'' - (2x+3)y' - 12y = 6x$. (8)

(ii) Solve by the variation of parameters method $\frac{d^2y}{dx^2} + y = \sec x$. (8)

12. (a) (i) Find the equation of the image of the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{4}$ in the plane $2x + y + z = 6$. (8)

(ii) Find the length of the shortest distance between the pair of lines $\frac{x-2}{2} = \frac{y+1}{3} = \frac{z}{4}$; $2x + 3y - 5z - 6 = 0 = 3x - 2y - z + 3$. (8)

Or

(b) (i) Find the equation of the tangent plane of the sphere $x^2 + y^2 + z^2 - 4x + 2y - 6z + 5 = 0$ which are parallel to the plane $2x + 2y - z = 0$. Find also their point of contact. (8)

(ii) Find the equation of the cone whose vertex is $(3, 1, 2)$ and the base curve is $2x^2 + 3y^2 = 1$; $z = 1$. (8)

13. (a) (i) If the center of curvature of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at one end of the minor axis lies at the other end, then prove that the eccentricity of the ellipse is $\frac{1}{\sqrt{2}}$.

(ii) Prove that the evolute of the tractrix $x = a(\cos\theta + \log[\tan(\theta/2)])$ and $y = a\sin\theta$ is a catenary.

Or

(b) (i) Find the envelope of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a and b are connected by the relation $a^2+b^2=c^2$, c being a constant.

(ii) Find the evolute of the rectangular hyperbola $xy=c^2$.

14. (a) (i) If $u=\tan^{-1}\left[\frac{x+y}{\sqrt{x}+\sqrt{y}}\right]$ show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{4}\sin 2u$. (8)

(ii) Find the minimum value of $x^2+y^2+z^2$ given that $ax+by+cz=p$. (8)

Or

(b) (i) If $u=e^{xy}$, show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{1}{u}\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right]$. (8)

(ii) Expand $e^{-x}\log y$ as a Taylor series expansion about $x=0$ and $y=1$ upto third order terms. (8)

15. (a) (i) Solve $(D^2+D)y=x\cos x$. (8)

(ii) Solve the following simultaneous equation $\frac{dx}{dt}+y=\sin t$, $x+\frac{dy}{dt}=\cos t$ given $x(0)=2$, $y(0)=0$. (8)

Or

(b) (i) Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}5 & 4\\ 1 & 2\end{bmatrix}$. Also compute A^{-1} and A^4. (8)

(ii) Diagonalise the matrix $A=\begin{bmatrix}2 & 1 & -1\\ 1 & 1 & -2\\ -1 & -2 & 1\end{bmatrix}$. (8)

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